

# On UltraViolet effects in protected inflationary models

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Inflationary models are usually UV sensitive. Several mechanism have been proposed to protect the necessary features of the potential, and most notably (softly broken) global symmetries as shift-symmetry. We show that, even in presence of these protecting mechanisms, the models maintain a serious UV-dependence. Via an improved effective theory analysis, we show how these corrections could significantly affect the duration of inflation, its robustness against the choice of initial conditions and the regimes that make it possible.

Inflation is a period of accelerated expansion of the universe conjectured to set up suitable initial conditions for the Hot Big Bang, alleviating its fine-tuning issues [1, 2]. It also provides initial conditions (the field perturbations) for structure formation. When quantized, these structure seeds are natural and not fine tuned [3].

The long debate on fine-tuning and on the effects of higher-energy physics [1, 2, 4–6] points to a UV dependence of inflationary models, which are typically effective field theories. Unfortunately, our candidate UV-complete theories are not sufficiently developed to give definite answers. Effective theory studies beyond the lowest order are thus necessary, notwithstanding the limitations of a bottom-up approach. The onset of inflation has been studied in various models [4, 6], and mechanisms have been proposed to make the fine-tuning of the effective potential more natural [2].

We shall however show that, even in the presence of such mechanisms, the models retain a serious UV-dependence, whose effects have not been studied systematically. Via an improved effective-theory analysis, we shall investigate quantitatively, and yet somewhat generally, how it can considerably affect dynamics, duration and onset of inflation.

We conservatively root our work in the present well-established theoretical framework, leaving for the future the analysis in specific UV completions.

To be definite, we shall focus on scalar field models of inflation driven by the potential energy, in particular single-field ones<sup>1</sup>, which are among the most studied and in very good agreement with observations

## INFLATION, PROTECTING MECHANISMS AND RESIDUAL UV SENSITIVITY

According to observations, the early universe is well described by a flat Friedman-Robertson-Walker metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 d\vec{x}^2, \quad H = \frac{\dot{a}}{a}, \quad (1)$$

where  $a$  is the scale factor,  $H$  is the Hubble rate and dots indicate time derivatives.

Accelerated expansion requires pressure  $p$  and energy density  $\rho$  such that  $p < -\rho/3$ . For the typical minimally coupled, canonically normalized scalar field, with action

$$S = - \int d^4x \left[ \frac{1}{2} g^{\mu\nu} \nabla_\mu \nabla_\nu \phi + V(\phi) \right], \quad (2)$$

accelerated expansion is reliably obtained if

$$\rho, -p \sim V, \quad \epsilon_v = \frac{M_{Pl}^2}{2} \left( \frac{\partial_\phi V}{V} \right)^2 \ll 1, \quad |\eta_v| = M_{Pl}^2 \left| \frac{\partial_\phi^2 V}{V} \right| \ll 1, \quad (3)$$

where  $M_{Pl} = (8\pi G)^{-1/2}$  is the reduced Planck mass. Evidently one needs to control quantum corrections in the potential. For example, any correction  $\delta V \sim V_{\text{tree}} \frac{\phi^2}{M_{Pl}^2}$  would lead to a change  $\delta\eta_v = O(1)$ , easily violating the bound on  $\eta_v$ . This is the famous “ $\eta$ -problem”.

Typical protection mechanisms make corrections to the potential naturally small exploiting exact or approximate symmetries to control their Wilson coefficients. Supersymmetry or global symmetries are typical choices<sup>2</sup>. However, supersymmetry has been shown not to generally solve the  $\eta$ -problem [2, 11], since it is necessarily broken at least at the scale of inflation.

A widely studied global symmetry is (softly broken) shift-symmetry  $\phi \rightarrow \phi + \text{constant}$  [7]. Global symmetries are possibly unnatural from the UV perspective, as they are expected to be broken by quantum gravity. Some ways out are, for example, making shift-symmetry

<sup>1</sup> Field redefinitions can trade terms between potential and kinetic parts of the action, making their distinction somewhat arbitrary. However, certain protecting mechanism (for example shift-symmetry) allow to distinguish them thanks to their transformation properties.

Note also that UV-sensitivity occurs in models driven by modified kinetic terms, as  $k$ -inflation [8, 9], and our analysis could be extended there. However, usually protecting mechanisms are less studied in those cases, and sometimes obstacles to UV completion appear [10]. We shall mention such models when relevant.

<sup>2</sup> We will mention the case of symmetries such as Galilean ones in footnote 4. They lead to Galilean genesis rather than inflation.

discrete and relating it to UV gauge symmetries, or realizing it non-linearly [12]. Despite these possible difficulties, (approximate) shift-symmetry is widely used in model building and thus an interesting example. Our results can be extended to other scenarios.

The approximate global symmetry reduces the Wilson coefficients of terms involving powers of the field, but does not constrain terms involving only field derivatives [13]. This remaining UV dependence is potentially serious. Higher-derivative corrections affect inflationary perturbations (via their dispersion relations)[14–16], but, as we shall see, also the background inflation, altering its duration, dependence on initial conditions and predictions.

Many candidate UV-complete theories possess distinctive high-derivative structures. A full answer can be given only when the complete UV theory is known, but an effective theory analysis is reliable when a perturbative expansion is possible. In this sense, and to make the work technically manageable, we begin by considering a generic inflaton Lagrangian, whose dependence on derivatives is truncated to first derivatives of the field:

$$\mathcal{L} = -M_f^4 P(\phi, X) - V(\phi), \quad X = \frac{g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi}{M_f^4}, \quad (4)$$

where  $M_f$  is the natural cutoff. This truncation must be consistent, as we shall discuss. Moreover, the analysis should include inhomogeneous initial conditions, but for the moment we study homogeneous ones. In fact, even with these limitations we shall point out relevant effects.

There are two important points. *First*, our classical analysis requires  $M_f < M_{\text{Pl}}$ . *Second*, in the absence of insights from the UV theory, the low-energy theory only allows imposing generic initial conditions up to the largest allowed values close to the cutoff  $M_f$ , and accounting for a soft breaking,

$$|\partial\phi|^2 \lesssim M_f^4, \quad V < M_f^4 \quad (5)$$

Finally, we recall that the system given by (4) and the Einstein-Hilbert action evaluated on the ansatz (1) is Hamiltonian, hence Liouville's theorem applies and no canonical standard attractor exist. However, inflationary attractors appear using certain coordinates [5]. This is the physical point: we describe the evolution of physically relevant variables. In our case they will be those we use to write the effective theory:  $\phi$  and  $\dot{\phi}$ .

## INFLATIONARY DYNAMICS WITH THE STANDARD LAGRANGIAN

The dynamical system obtained from (2) and the Einstein-Hilbert action with the ansatz (1), is

$$\begin{cases} \dot{\phi} = y \\ \dot{y} = -\sqrt{3} \frac{\sqrt{\frac{y^2}{2} + V(\phi)}}{M_{\text{Pl}}} y - \partial_\phi V(\phi) \end{cases}, \quad (6)$$

using the Hamiltonian constraint  $H^2 = \frac{1}{3M_{\text{Pl}}^2} \left( \frac{y^2}{2} + V \right)$ .

The system is effectively two-dimensional, so that dynamical chaos, which would make the dependence on initial conditions dramatic, cannot occur. The only fixed points of the system (6) are of the form  $(\phi, y) = (\phi_c, 0)$  with  $\partial_\phi V|_{\phi_c} = 0$ . Points at infinity do not occur because of the bounds (5). At the fixed points, the solution is flat (for  $V(\phi_c) = 0$ ) or de Sitter-like with constant  $H = \sqrt{\frac{3V(\phi_c)}{M_{\text{Pl}}^2}}$ . One can easily see that the fixed points are asymptotically stable if  $(\partial_\phi^2 V(\phi))|_{\phi_c} > 0$  and unstable if  $(\partial_\phi^2 V(\phi))|_{\phi_c} < 0$ . However, these attractors are not the physically interesting ones: a pure de Sitter expansion would not yield the necessary density perturbations to account for structures, nor it will have an end<sup>3</sup>.

In fact, the inflationary solutions arise in a different way. We describe now a method to quickly find them, and determine their attractor behavior without obtaining the whole phase-space diagram.

**A)** We first determine the regions  $\mathcal{D}$ ,  $\mathcal{I}$ ,  $\mathcal{S}$  in  $(\phi, y)$ -space where  $\dot{\phi}$  respectively decreases, increases and is stationary. For the system (6), they are individuated by  $\frac{\sqrt{3}y}{M_{\text{Pl}}} \sqrt{\frac{y^2}{2} + V} + \partial_\phi V$  respectively positive, negative or zero.

**B)**  $\mathcal{S}$  is thus constituted of curves  $y = y_*(\phi)$  satisfying  $\frac{\sqrt{3}y}{M_{\text{Pl}}} \sqrt{\frac{y^2}{2} + V} + \partial_\phi V = 0$ . They are *not* solutions of the system (6), but integrating  $\dot{\phi} = y_*(\phi)$  and inserting it in (6), we see that the parts of  $y_*(\phi)$  where the slow-roll condition

$$|y_* \partial_\phi y_*| \ll |y_*| \sqrt{\frac{3}{M_{\text{Pl}}^2} \left( \frac{y_*^2}{2} + V(\phi) \right)} \quad (7)$$

holds, are *good approximations* of solutions. Inflationary trajectories occur in particular when  $\frac{y_*^2(\phi)}{2} \ll V(\phi)$ .

**C)** Given its definition,  $\mathcal{S}$  contains the fixed points of the system. Thus, since  $\mathcal{S}$  separates regions  $\mathcal{D}$  and  $\mathcal{I}$ , and the flow draws from  $\mathcal{D}$ ,  $\mathcal{I}$  on  $\mathcal{S}$ , if there is only a global asymptotic attractor then the approximate inflationary solutions in  $\mathcal{S}$ , see B), are essentially global attractors.

**C')** If there are instead multiple locally attractive fixed points, the inflationary solutions ending on them are at least attractive for a local set of initial conditions.

**Duration and onset of inflation.** Even with attractor solutions, the inflationary phase must still comply with observations and provide correct initial conditions for the Hot Big-Bang. In particular, the inflationary attractors must be reached early enough and allow a suitable duration of inflation [17].

This is partly model-dependent, but one can discuss how fast solutions where the kinetic term is initially sizable reach the inflationary regime. For the standard

<sup>3</sup> Ghost inflation [9], which is driven by special kinetic terms, allows scalar perturbations in pure de Sitter, but it has tachyons in other backgrounds, and might not admit unitary UV completion.

system (6) one finds that all solutions where  $\dot{\phi}^2 \gg V(\phi)$ ,  $M_{\text{Pl}}|\partial_\phi V(\phi)|$  will damp exponentially their kinetic energy density [4], approximately as

$$\dot{\phi} \sim \pm M_f^2 e^{-\sqrt{\frac{2}{3}} \frac{|\phi - \phi_{\text{in}}|}{M_{\text{Pl}}}} \quad (8)$$

This indicates that inflation would take place quite rapidly, as soon as  $|\Delta\phi| = |\phi - \phi_{\text{in}}| > M_{\text{Pl}}$ . While this argument is not relevant for small-field models (where the field excursion is sub-Planckian), it seems to make the situation more favorable in large-field models. We shall see shortly how this changes when corrections are taken into account.

### DYNAMICS WITH HIGHER-ORDER DERIVATIVE CORRECTIONS

After using the Hamiltonian constraint  $H^2 = \frac{\rho}{3M_{\text{Pl}}^2}$  the field equations derived from the action (4) read

$$\begin{cases} \dot{\phi} = y \\ \dot{y} = -\sqrt{3}c_s^2 \frac{\sqrt{2P_{,X}y^2 + M_f^4 P + V}}{M_{\text{Pl}}} y + \frac{M_f^4}{2} \frac{\partial_\phi \rho}{\partial_X \rho} \end{cases} \quad (9)$$

where commas indicate partial derivatives and

$$\rho = 2y^2 P_{,X}(\phi, X) + M_f^4 P(\phi, X) + V(\phi), \quad (10)$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} - 2P_{,XX}y^2 M_f^{-4}}. \quad (11)$$

The corrections could lead to new attractors, making suitable inflationary solutions harder to approach without fine-tuning of initial conditions. Moreover, even in absence of new undesirable attractors, they could affect the features of inflationary solutions, for instance leading to too short a period of inflation or to signatures incompatible with observations.

A few basic and necessary physical conditions suffice to constrain  $P(\phi, X)$  and get concrete results concerning these questions. As the complete theory is unknown, we can only consider very basic conditions: 1) a stable and causal system, 2) a well-posed Cauchy problem for the field dynamics and 3) a self-consistent theory.

Stability and causality require at least the weakest energy conditions: the null energy one (NEC)<sup>4</sup> [10, 18]. From the action (4),

$$T_{\mu\nu} \equiv 2P_{,X}\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}(M_f^2 P + V), \quad (12)$$

and the NEC states that for any null vector  $n^\mu$

$$T_{\mu\nu}n^\mu n^\nu \geq 0 \Rightarrow P_{,X} \geq 0. \quad (13)$$

Other conditions follow from the covariant field equation

$$\left(P_{,X}g^{\mu\nu} + 2\frac{\nabla^\mu\phi\nabla^\nu\phi}{M_f^4}P_{,XX}\right)\nabla_\mu\nabla_\nu\phi + P_{,X\phi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{(M_f^4 P + V)_{,\phi}}{2} = 0. \quad (14)$$

This would be generally degenerate if  $P_{,X} = 0$ , so one further requires

$$P_{,X} > 0. \quad (15)$$

Writing the principal part of eq. (14) in terms of the “effective” inverse metric  $G^{\mu\nu} = g^{\mu\nu} + 2\frac{P_{,XX}}{P_{,X}}\frac{\nabla^\mu\phi\nabla^\nu\phi}{M_f^4}$ , one can see that the equation is hyperbolic, and thus defines a well-posed Cauchy problem [10, 20], if

$$P_{,X} + 2\frac{\nabla^\mu\phi\nabla_\mu\phi}{M_f^4}P_{,XX} > 0. \quad (16)$$

Moreover, the lightcone of the metric  $G_{\mu\nu}$  is inside or on the light cone of the metric  $g_{\mu\nu}$  if

$$\frac{P_{,XX}}{P_{,X}} \leq 0 \Rightarrow P_{,XX} \leq 0, \quad (17)$$

where at the end we have used (15). If (17) is satisfied no superluminal motion is possible.

These basic physical requirements have nothing to do with inflation in itself, but they will lead to important consequences for it. Let us stress that while the conditions from the NEC and well-posed Cauchy problem are rather basic, (17) seems not as fundamental. However, there is an obstruction to UV completion if it is violated [10], thus we adopt it.

Finally, a softly broken shift-symmetry also constrains the dependence on  $\phi$  of  $P(\phi, X)$  and  $V(\phi)$ . Indeed, from the almost conserved Noether current, one finds that  $|\partial_\phi V|, M_f^4|\partial_\phi P| \propto \lambda_{sb}^n$ ,  $n$  positive, with  $\lambda_{sb}(\text{'s}) \ll 1$  the symmetry breaking parameter(s).

### Self-consistency of the theory

In the effective theory truncated to finite order in derivatives, the only trustworthy solutions are those where higher derivatives are subdominant [21]. However, a generic action as (4) would typically admit runaway solutions not satisfying this condition. This is unsuitable to study onset and robustness of inflation, as one would not be able to consider generic initial conditions, according to the bounds (5), and trust all solutions. Hence, one must make sure that higher-order derivatives are subdominant for the relevant space of solutions, up to the generic initial conditions (5), close to the proper cutoff  $|X| \lesssim 1$  (but within the validity limits of our effective theory). The consequences of this have typically been neglected in the literature [4, 6].

<sup>4</sup> There have been various attempts to construct reliable NEC-violating models in Galileon and ghost condensate scenarios, but no totally healthy and under control model has yet appeared [19].

Minimal conditions on the action that ensure this behavior can be found by looking at the field equations (9). Second derivatives can be ignored up to  $|X| \lesssim 1$  provided

$$c_s^2|_{|X| \lesssim 1} \ll 1 \Rightarrow -\frac{P_{,XX}}{P_{,X}}|_{|X| \lesssim 1} \gg 1. \quad (18)$$

Note that this is precisely what happens with the DBI action [22]. However, that case is peculiar since in some of the models inflation itself occurs near the cutoff region, leading to strong constraints on the models.

We demand instead that  $c_s^2 \ll 1$  *not* during inflation, but around  $|X| \lesssim 1$ , to avoid strong coupling of inflationary perturbations and large non Gaussianities, which would generally invalidate the inflationary model(s) [15].

For the system (9), the condition  $c_s^2|_{|X| \lesssim 1} \sim 0$  implies that the trajectories leave the region  $|X| \lesssim 1$  *slowly*, since  $\dot{y} \sim 0$ . As we shall see, this affects inflation.

### Inflation and high-derivative corrections

We list the most relevant results, in order of concern.

*i) Attractors and fixed points.* Because of conditions (15) and (16), the only fixed points of the high-energy corrected system (9) are

$$\{(\phi, y) = (\phi_c, 0) \mid \partial_\phi \rho(\phi_c, 0) = 0\}. \quad (19)$$

These are equivalent to the fixed points of the standard system (6) since  $\partial_\phi \rho|_{(\phi_c, 0)} = \partial_\phi V|_{\phi_c}$ . This is comforting, as it excludes the possibility for the solutions to end up in different attractors and goes in the direction of supporting the standard dynamics.

New exact  $y$ -stationary points would require  $\partial_X P|_{(\phi, y)_{\text{new}}} = 0$  as well as  $\partial_\phi \rho|_{(\phi, y)_{\text{new}}} = 0$ , but this is prevented by the well-posedness of the field equations, see in particular eq. (15). Approximate  $y$ -stationary cases ( $(\partial_X P, \partial_\phi \rho)|_{(\phi, y)_{\text{new}}} \ll 1$ ) are possible, but would generally possess unsuitable features (such as a sound speed  $c_s^2 \ll 1$  for the perturbation, which would have left potentially observable imprints [15, 16]).

Approximate inflationary solutions can be found with the method described for the standard action.

The region  $\mathcal{S}$  (where  $\dot{\phi}$  is stationary) is now defined by

$$-\sqrt{3}c_s^2 \frac{\sqrt{2P_{,X}y^2 + M_f^4 P + V}}{M_{\text{Pl}}} y + \frac{M_f^4}{2} \frac{\partial_\phi \rho(\phi)}{\partial_X \rho(\phi)} = 0, \quad (20)$$

and the regions  $\mathcal{D}/\mathcal{I}$  of decreasing/increasing  $\dot{\phi}$ , occur where the left-hand side of (20) is negative/positive.

The curves  $y_*(\phi) \in \mathcal{S}$  approximate actual solutions when

$$|y_* \partial_\phi y_*| \ll c_s^2 \sqrt{3} \frac{\sqrt{2P_{,X}y_*^2 + M_f^4 P + V}}{M_{\text{Pl}}} |y_*|, \quad (21)$$

which replaces condition (7) of the standard scenario. These approximate solutions are inflationary when

$2P_{,X}y^2 + M_f^4 P \ll V$ . As before, the global or local attractor nature of the inflationary solutions depends on that of the fixed points (19), which are also part of  $\mathcal{S}$ .

*ii) Onset of inflation.* To start potential-driven inflation one typically needs lower values of  $\dot{\phi}^2$  than when using the standard action, since the kinetic energy density increases more rapidly with  $\dot{\phi}^2$  than the standard case because of conditions (16), (17), (18), while shift-symmetry makes it weakly dependent on  $\phi$ . Hence, solutions starting with generic initial conditions may reach the inflationary regime too late. Next we present yet another effect that contributes even more to this issue.

*iii) Duration of inflation.* The most relevant results concern the duration of inflation and related predictions/implications.

The consistency conditions ensuring that higher derivatives are subdominant in the effective theory truncated at finite order in derivatives imply that solutions flow more slowly out of the region  $|X| \lesssim 1$ , see eq. (18) and comments. Indeed, eqs. (18), (17) and the weak dependence of  $P$  on  $\phi$  lead to an approximate solution for large  $y$  and  $V$  subdominant in the energy density:

$$\text{sign}(y) \left[ P_{,X} \left( -\frac{y^2}{M_f^4} \right)^{-\frac{1}{2}} - P_{,X}(-1)^{-\frac{1}{2}} \right] \propto \frac{\sqrt{3}}{M_{\text{Pl}}} (\phi - \phi_{\text{in}}). \quad (22)$$

They also show that  $P_{,X} \left( -\frac{y^2}{M_f^4} \right)^{-\frac{1}{2}} - P_{,X}(-1)^{-\frac{1}{2}} > -\log\left(\frac{|y|}{M_f}\right)$  (indeed, using eq. (18), one sees that typically the right-hand side grows much more rapidly than the logarithm when  $|y|$  decreases from  $\lesssim M_f$ ). Thus, comparing eq. (22) and the logarithm of (8), one can see that low values (inflationary regime) of  $y = \phi$  are typically attained more slowly than the exponential damping found when higher-derivative corrections are not taken into account. This slower motion would extend beyond the cutoff region, depending on the explicit form of  $P(\phi, X)$ .

Due to this slower damping of the kinetic energy, the inflationary attractor would be reached generally later than when derivative corrections are not accounted for. The duration of inflation could then be insufficient to provide the desired effects on the subsequent universe evolution [1, 2], or lead to observable signatures [23].

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